

B. Math. I
Semestral Examination
Algebra - I

- All questions are compulsory.
- All questions carry equal marks.

- (1) Let p be a prime number. Show that any group of order p^2 is abelian.
- (2) Let G be a finite group and let H be a Sylow p -subgroup of a group G . Show that $N(N(H)) = N(H)$.
- (3) Describe the structure of the ring $\frac{\mathbb{Z}[i]}{(2+i)}$.
- (4) What is a nilradical of $\frac{\mathbb{Z}}{32\mathbb{Z}}$?
- (5) Find all ideals I of the ring $R = \frac{\mathbb{Z}}{12\mathbb{Z}}$. In each case compute R/I , i.e. find a known ring to which the quotient ring is isomorphic.
- (6) Solve $X^2 - X + 6$ in $\frac{\mathbb{Z}}{12\mathbb{Z}}$.
- (7) Let R be commutative ring with unity. Let I be an ideal of R . Describe the ideals in the quotient ring R/I . Justify your answer.